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Title: An Introduction to Multiplicity Counting

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An Introduction to Multiplicity Counting

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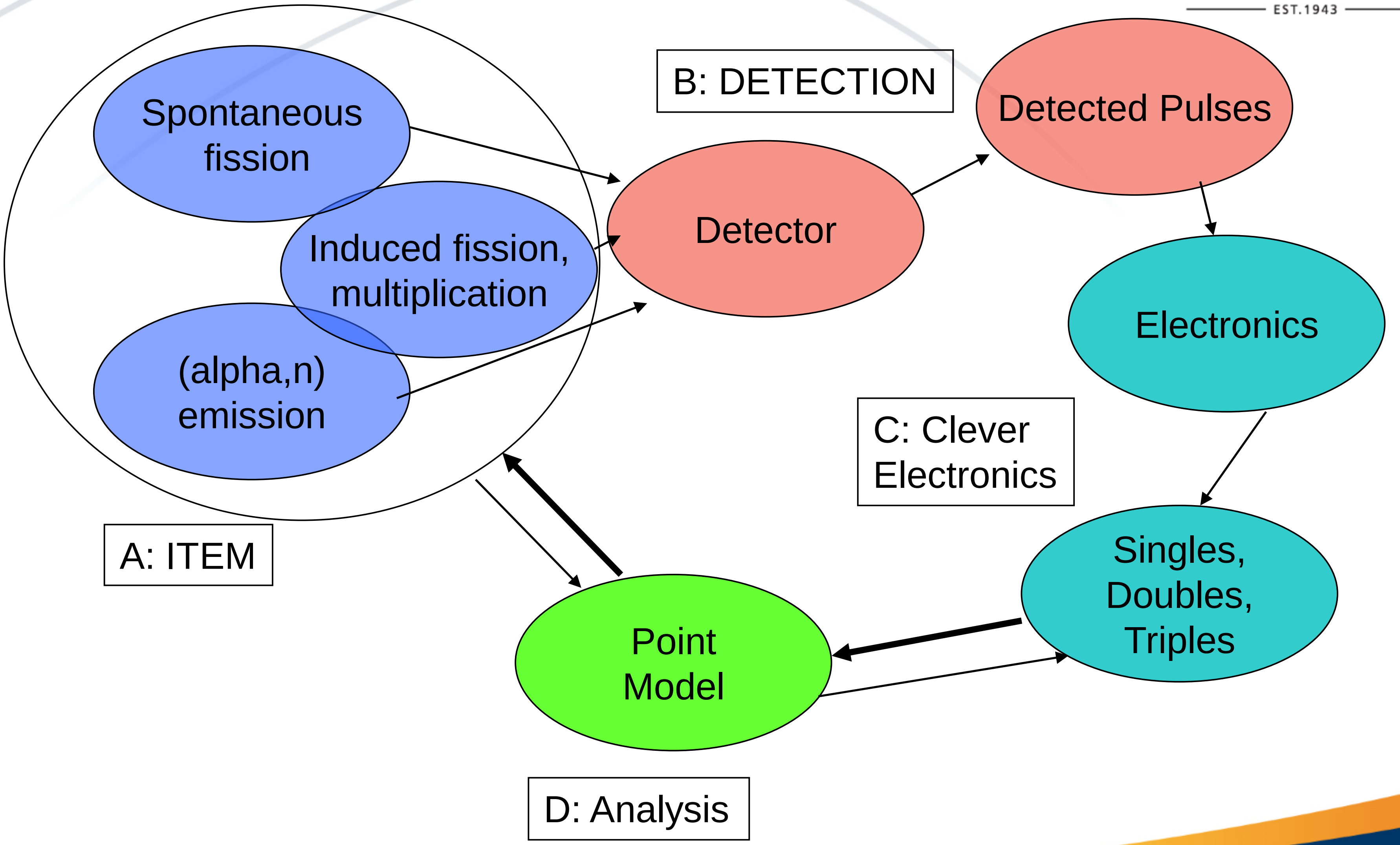
The Problem with Standard Coincidence Counting

- There are 3 principal unknowns in neutron counting:

  ^{240}Pu-effective mass, α, and M.

- Standard Coincidence Counting: Provides only ***2*** pieces of measured information, singles and doubles (or totals and coincidences)
  - To obtain an accurate assay, one must know a lot about the sample.
  - If the assumed information is incorrect, large errors can occur.
- Neutron Multiplicity Counting: ***3*** pieces of measured information are used with a mathematical model to deduce an assay that is far superior for most impure materials.

Multiplicity counting process



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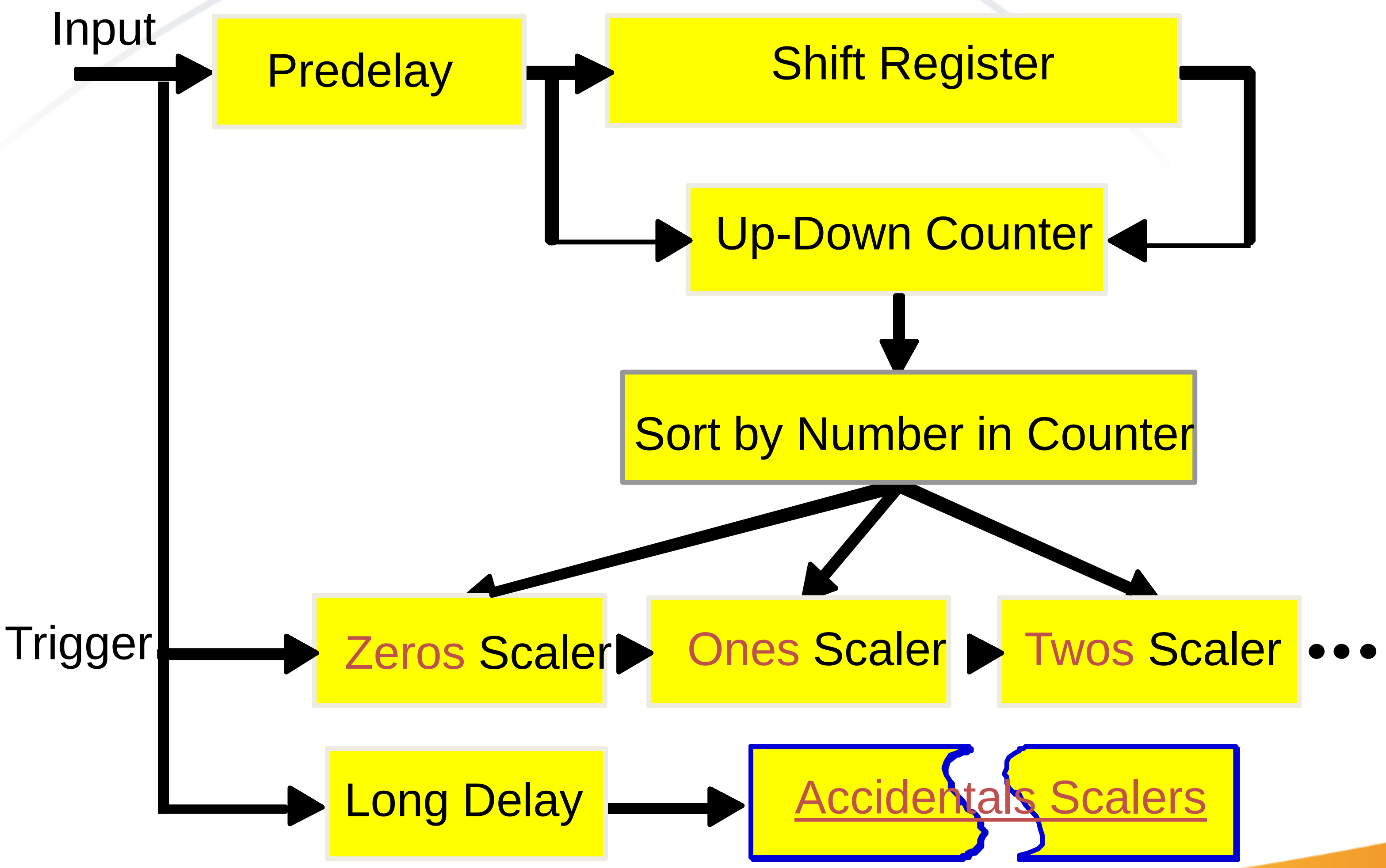
Multiplicity Shift Register



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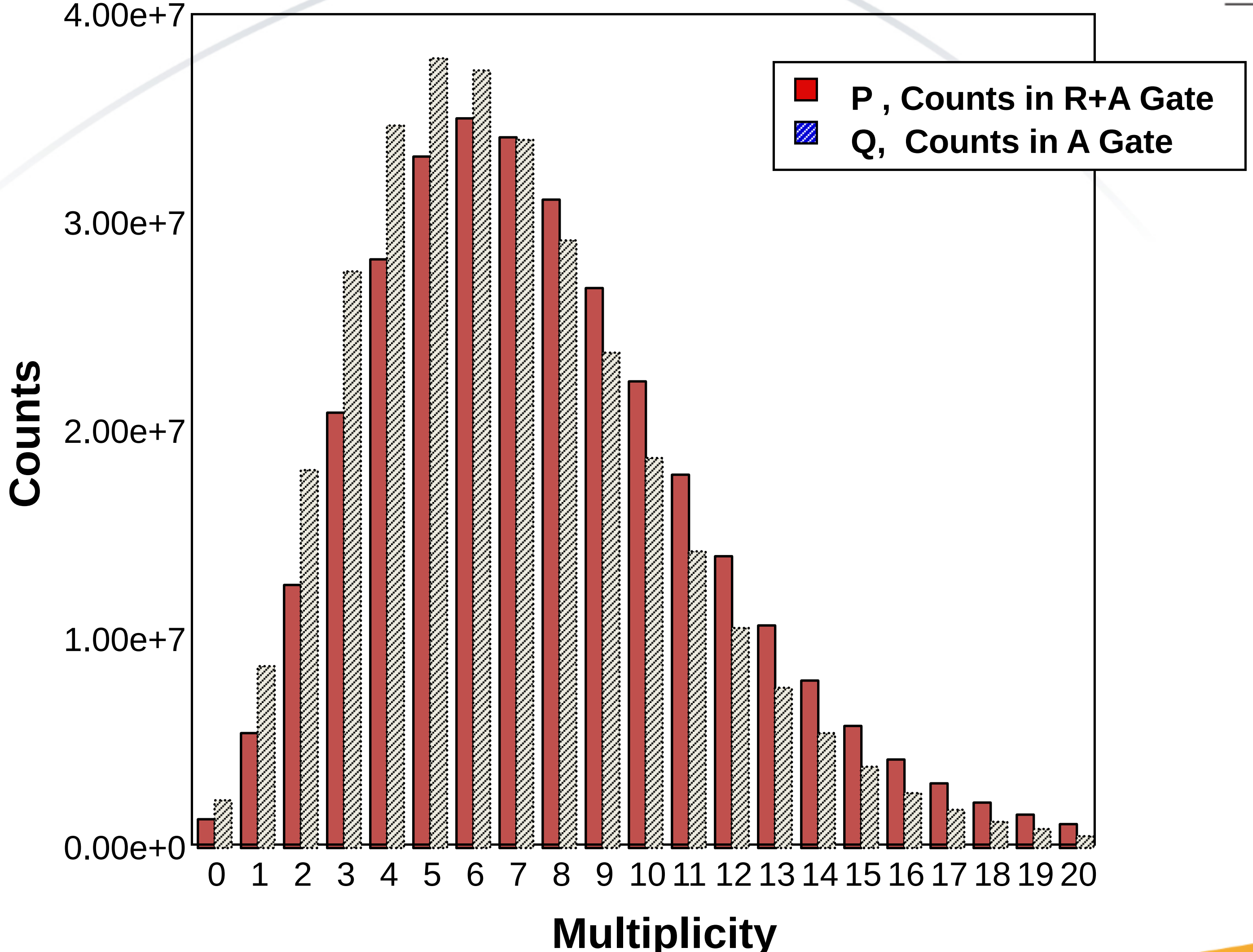
Multiplicity Distribution - 3.8 kg Pu Metal



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Count Rates from the Multiplicity Shift Register

- $\mathbf{P_n}$ = # times a multiplicity of n is counted in the **R + A** gate

- $\mathbf{Q_n}$ = # times a multiplicity of n is counted in the **A** gate

- We get singles, doubles, and triples information from a combination of the moments of these distributions.

Singles

- The singles rate is the sum of all of the single neutrons detected.

$$\textbf{measured singles} = \sum_{n=0}^{N} Q_n$$

- The singles rate is also sometimes referred to as the “Zeroth” moment of the accidentals distribution.

Doubles

- The doubles are the difference in the 1st moments of the multiplicity distributions in the R+A and A Gates.

$$\text{measured doubles} = \sum_{n=1}^{N} nP_n - \sum_{n=1}^{N} nQ_n$$

- The doubles obtained this way are <u>equivalent</u> to the real coincidences obtained with a standard shift register circuit – this provides a useful diagnostic for multiplicity shift register operation.

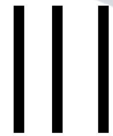
Triples

- The formula for calculating triples is intuitively much harder because the information in the R+A and A gates is correlated.
- The triples are the difference in the 2nd moments *minus* a cross correlation term that depends on the doubles.

$$\text{measured triples} = \sum_{n=2}^{N} \frac{n\,(n-1)}{2} P_n - \sum_{n=2}^{N} \frac{n\,(n-1)}{2} Q_n$$

$$- \frac{\sum_{n=1}^{N} n Q_n}{\sum_{n=0}^{N} Q_n} \left(\sum_{n=1}^{N} n P_n - \sum_{n=1}^{N} n Q_n \right)$$

Shift Register Comparison



3 2 1

Multiplicity Shift Register

0	1
1	1
2	1

$$\text{Doubles} = \sum_{n=0}^{2} nPn =$$

$$0*1+1*1+2*1=3$$

$$\text{Triples} = \sum_{n=0}^{2} \frac{n(n-1)}{2} Pn$$

$$= \frac{0(0-1)}{2}.1 + \frac{1(1-1)}{1}.1$$

$$+ \frac{2(2-1)}{2}.1 = 1$$

Regular Shift Register

Doubles = 0+1+2 = 3

Total Neutron Rate or “Singles”

source neutrons = s. f. + (α ,n) neutrons

$$= (1+\alpha)\,\nu_{s1}\,F$$

all emitted neutrons = M * source neutrons

all detected neutrons = ε * all emitted neutrons

Singles : $S = \varepsilon M (1+\alpha)\,\nu_{s1}\,F$

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Real Coincidence Rate or “Doubles”

Doubles from spontaneous fission = ν_{s2} F

But there are no doubles from (α,n) unless the sample is multiplying.

So an equation for doubles needs three components:

1. doubles from s.f. source doubles,
2. doubles from i.f. of s.f. source singles, and
3. doubles from i.f. of (α,n) neutrons.

$$D = (f_d / 2)\ (\varepsilon M)^2\ F \{ \nu_{s2} + [(M-1)/(\nu_{i1}-1)]\ \nu_{s1}(1+\alpha)\,\nu_{i2} \}$$

Triples

It is not so simple to write down an equation for triples.

$$T = \frac{F\varepsilon^3 f_t M^3}{6}\left\{\nu_{s3} + \left(\frac{M-1}{\nu_{i1}-1}\right)\left[3\nu_{s2}\nu_{i2} + \nu_{s1}(1+\alpha)\nu_{i3}\right] + 3\left(\frac{M-1}{\nu_{i1}-1}\right)^2 \nu_{s1}(1+\alpha)\nu_{i2}^2\right\}$$

Changing Count Rate to Mass

- To determine the total Pu mass the isotopic values need to be known.

$$^{240}Pu_{eff}\% = 2.52\left(^{238}Pu\%\right) + {}^{240}Pu\% + 1.68(^{242}Pu\%)$$

$$m_{Pu} = \frac{^{240}Pu_{eff}(g)}{2.52\left(^{238}Pu\right) + {}^{240}Pu + 1.68\left(^{242}Pu\right)}$$

- Example: 200 g of Pu with ^{238}Pu=2%, ^{240}Pu=24% and ^{242}Pu=6%

$$^{240}Pu_{eff}\% = 2.52 \text{ x } 2\% + 24\% + 1.68 \text{ x } 6\% = 39.12\%$$

$$^{240}Pu_{eff}\ g = 39.12\% \text{ x } 200 \text{ g} = 78.24 \text{ g}$$

The Point Model Equations

$$S = F \varepsilon M \nu_{s1}(1+\alpha)$$

$$D = F(f_d/2)(\varepsilon M)^2 \{\nu_{s2} + [(M-1)/(\nu_{i1}-1)]\, \nu_{s1}(1+\alpha)\nu_{i2}\}$$

$$T = F(f_t/6)(\varepsilon M)^3 \{\nu_{s3} + [(M-1)/(\nu_{i1}-1)][3\,\nu_{s2}\nu_{i2} + \nu_{s1}(1+\alpha)\,\nu_{i3}] + 3[(M-1)/(\nu_{i1}-1)]^2\, \nu_{s1}(1+\alpha)\nu_{i2}^2\}$$

where:

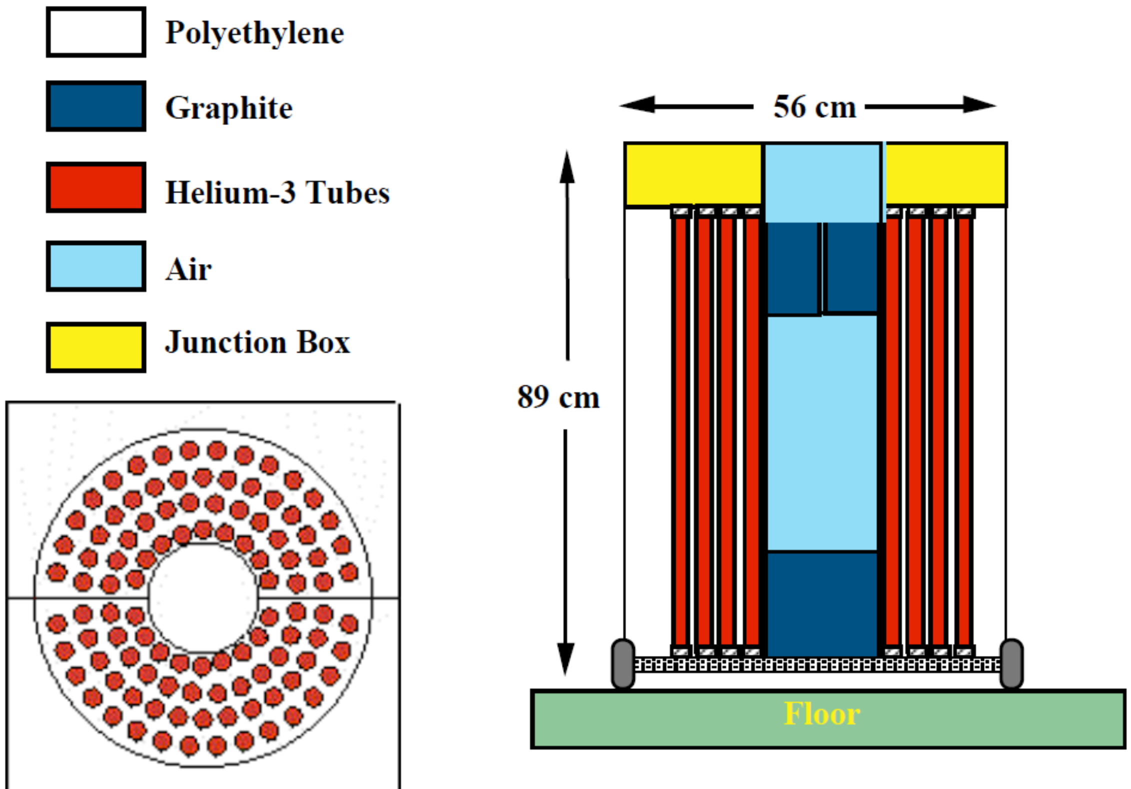
- ε = detection efficiency
- f_d = fraction of doubles in the coincidence gate
- f_t = fraction of triples in the coincidence gate
- **F** = spontaneous fission rate = 473.5 n/s/g * effective PU-240 mass
- ν_{n1} = average number of neutrons produced per fission event (n=s -- spontaneous fission, n= i -- induced fission)
- ν_{n2} = average number of neutron pairs produced per fission event
- ν_{n3} = average number of neutron "triplets" produced per fission event

α = ratio of (alpha, n) neutron rate to the spontaneous fission rate

M = fission multiplication

Mini-Epithermal Neutron Multiplicity Counter (miniENMC)

- High-efficiency neutron counter (61.8%)
- Passive neutron coincidence and multiplicity counting
- Made for dirty scrap and bulk items
- Little a-priori knowledge of item required



104 Tubes

Sample Cavity is 16 cm x 38 cm and is Cadmium Lined